2.1 The Real Numbers and Absolute Values

Natural Numbers (counting numbers 1, 2, 3 …)

Whole Numbers (0, Natural Numbers)

Integers (…, -3, -2, -1, and Whole Numbers)

Rational Numbers (Integers and any number that can be expressed in the form of $\frac{a}{b}$, where a and b are integers and b≠0, is a rational number.)

Terminating Decimal in the form of $\frac{3}{4}$, or 0.750

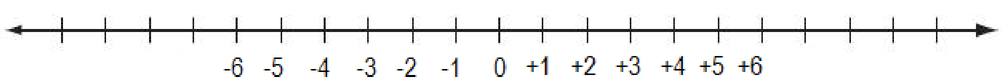
Repeating Decimal in the form of $\frac{1}{3}$, or 0.333 or $0.\bar{3}$

Irrational Numbers:

Non-repeating Decimal in the form of π=3.149…, $\sqrt{7} = 2.64575131 \ldots$

Real Numbers (Irrational and Rational)

<u>Ordering Numbers</u>



The **number line** represents the entire set of real numbers. Each point on the number line represents exactly one number. Each number in a set of real numbers corresponds to exactly one point on the number line. This allows the numbers on a number line to be ordered. Numbers on the number line increase from left to right.

The symbols used for ordering are:

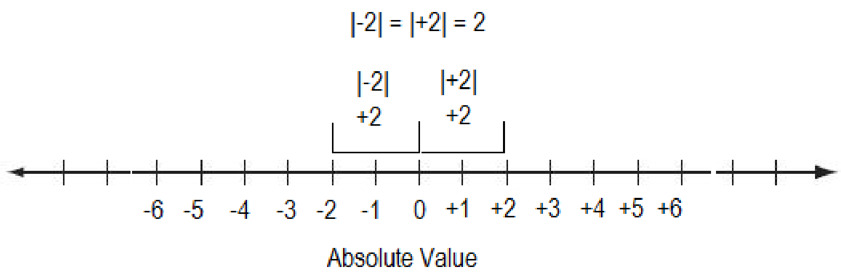
< less than	$\leq$ less than or equal to
> greater than	$\geq$ greater than or equal to
= equal to	

$a \leq b$ this expression reads…**a** is less than or equal to **b**.

When comparing two numbers on the number line (above), the number on the left will always be smaller and the number on the right will always be larger.

Definition of Absolute Value

The **absolute value** of a real number x is the distance from x to 0 on a number line. The symbol |x| means the absolute value of x.



Notice that a number and its opposite have the same absolute value. The absolute value of 2 and -2 are the same, 2. Each is 2 units from 0.

The absolute value of a positive number or 0 is the same as the number itself.
The absolute value of a negative number is the opposite of the number.

Examples: Simplify

a. |-7| b. |-9+5| c. |-(6-3)|

<u>Lesson 2.2 Adding Real Numbers</u>

<u>Rules for Adding Two Signed Numbers</u>

Addition of two numbers **<u>with like signs</u>**

1. Find the sum of the absolute values.
2. Use the sign common to both number

Examples A:

5+7= |5| + |7| = 5 + 7 = 12, Use the + sign (common to both), ans=+12.

-5-7= |5| + |7| = 5 + 7 = 12, Use the - sign (common to both), ans=-12.

Example B:

-3 + (-3), The expression reads "negative 3 plus negative 3".
Step 1: |-3| + |-3| = 3 + 3 = 6
Step 2: Negative sign is common, -6.
-3 + (-3) = -6

Addition of two numbers **<u>with unlike signs</u>**

1. Find the difference of the absolute values.

2. Use the sign of the number with the greater absolute value.

Example A:

5-7= |5| - |7| = 5 - 7 = -2, Use the – because |7|>|5|, ans=-2.

Or 5-7= |7| - |5| = 7 - 5 = 2, Use the – because |7|>|5|, ans=-2.

Example B:

-8 + 5, The expression reads "negative 8 plus five".

Step 1: |5| - |-8| = 5 – 8 = -3

Step 1: |-8| - |5| = 8 – 5 = 3

Step 2: |-8| is greater, use the negative sign, -3.

Identity Property for Addition

For all real numbers a, a + 0 = a and 0 + a = a.

Additive Inverse Property

For every real number a, there is exactly one real number -a such that a + (-a) = 0, and -a + a = 0.

Lession 2.3 Subtracting Real Numbers

Definition of Subtraction

For all real numbers a and b: $a - b = a + (-b)$

In words….."Subtract **b** from **a**" is equal to "find the opposite of **b** than add it to **a**."
Or "The difference of **b** and **a** is equal to the sum of **a** and the opposite of **b**."

Examples: Use the definition of subtraction to find each difference.

a. $3 - (-4)$
The opposite of -4 is 4.
Add 3+4=7.

b. $-5.6 - (-1.4)$
The opposite of -1.4 is 1.4.
Add $-5.6 + 1.4 = -4.2$

2.4 Multiplying and Dividing Real Numbers

Rules for Multiplying Two Signed Numbers

Multiplying Two Numbers With Like Signs

1. Find the product of the absolute values of the numbers.
2. Wire the product as a positive number.

(+) • (+) = (+) (-) • (-) = (+)

Multiplying Two Numbers With Unlike Signs

1. Find the product of the absolute vales of the numbers.
2. Write the produce as a negative number.

(+) • (-) = (-) (-) • (+) = (-)

Rules for Dividing Two Signed Numbers

Dividing Two Numbers With Like Signs

1. Find the quotient of the absolute values of the numbers.
2. Wire the quotient as a positive number.

(+) ÷ (+) = (+) (-) ÷ (-) = (+)

Dividing Two Numbers With Unlike Signs

1. Find the quotient of the absolute vales of the numbers.
2. Write the quotient as a negative number.

(+) ÷ (-) = (-) (-)÷ (+) = (-)

Identify Property for Multiplication

For all real numbers a, a • 1 = a and 1 • a = a.

Multiplicative Inverse Property (Reciprocal Property)

For every nonzero real number a, there is exactly one number $\frac{1}{a}$ such that

$$a \bullet \frac{1}{a} = 1 \text{ and } \frac{1}{a} \bullet a = 1$$

The number $\frac{1}{a}$ is called the **reciprocal or multiplicative inverse** of a.

Properties of Zero

Let a represent any real number.

1. The product of any real number and zero is zero.
   a • 0 = 0 and 0 • a = 0
2. Zero divided by any nonzero real number is zero.
   0 ÷ a = 0 or $\frac{0}{a}$ = 0, where a≠0.
3. Division by zero is undefined. (Division by zero is not possible.)

2.5 Properties of Mental Computation

Commutative Property of Addition

For all real numbers a and b: a + b = b + a

Commutative Property of Multiplication

For all real numbers a and b: a • b = b • a

Associative Property of Addition

For all real numbers a, b and c: (a + b) + c = a + (b + c)

Associative Property of Multiplication

For all real numbers a, b and c: (a • b) • c = a • (b • c)

The Distributive Property involves addition, subtraction, and multiplication. An important property of numbers is that multiplication can be distributed over addition and subtraction.

The <u>Distributive Property</u> of Multiplication Over Addition and Subtraction

For all real numbers a, b and c: a(b + c) = ab + ac and (b + c)a = ba + ca

and

For all real numbers a, b and c: a(b - c) = ab - ac and (b - c)a = ba - ca

$$a(b+c)=ab+ac$$

The arrows indicate the distribution property of multiplication over addition.

You can use the Commutative, Associative and Distributive Properties to rearrange expressions for quick mental computation.

Examples

i. 8(9.5) = 8(9.0 + 0.5) = (8)(9) + (8)(0.5) = 72 + 4 = 76.
ii. 2(2.5 + 5.9) resembles the left side of the expression a(b + c),
Where a=2, b=2.5 and c=5.9
Evaluate it by distributing, that is, (2)(2.5) + (2)(5.9), Solve it.
iii. (9xy-21xyz) resembles the right side of the expression ab + ac,
Where ab=9xy and ac=21xyz or ac=(-21xyz).
Evaluate it by factoring common terms, in this case the common terms are 3xy, that is, (3xy)(3-7z)

Opposite of Numbers and Expressions

You can find the opposite of a number by multiplying it by -1.
You can find the opposite of an expression by multiplying by -1.

Examples: Evaluate (-1)(7)=(-7)

(-1)(a + b), Distribute (-1)(a + b) = (-1)(a) + (-1)(b) = -a – b

(-1)(x-3), Distribute (-1)(x-3) = (-1)(x)-(-1)(3) = -x-(-3) = -x+3

Properties of Equality

Reflexive Property a = a (A number is equal to itself.)
Symmetric Property If a = b, then b =a.
Transitive Property If a =b and b=c, then a=c.
Substitution Property If a = b, than a can be replaced by b and b can replaced by a.

2.6 Adding and Subtracting Expressions

Warehouse Materials: 3x + 4
Retail Store: 5x + 2

We can combine these two expression into one expression 3x + 4 + 5x + 2.

In an expression, parts that are added together are called **like terms**. The **3x** and **5x** are like terms because they contain *the same form of the variable x*. The numerical part of an x-term, in this case **3 and 5**, is called the **coefficient**. The numbers 4 and 2, which are also like terms, are called **constants**.

The simplified expression is 8x + 6.

2.7 Multiplying and Dividing Expressions

Symbol	Read as	Mathematical Meaning	Value
3^1	3 to the 1st power	3	3
3^2	3 to the 2nd power, or squared	$3 \cdot 3$	9
3^3	3 to the 3rd power, or cubed	$3 \cdot 3 \cdot 3$	27
3^4	3 to the 4th power	$3 \cdot 3 \cdot 3 \cdot 3$	81

Symbol	Read as	Mathematical Meaning
x^1	x to the 1st power	x
x^2	x to the 2nd power, or squared	$x \cdot x$
x^3	x to the 3rd power, or cubed	$x \cdot x \cdot x$
$3x^4$	3 times x to the 4th power	$3(x \cdot x \cdot x \cdot x)$

In an expression such as x^4, x is called the **base** and 4 is called the **exponent**.

Examples: Simplify

a) $2x(3x-4)$, Distribute $(2x)(3x)-(2x)(4) = (6)(x \cdot x) - 8x = 6x^2 - 8x$
b) $(-4)(3x+4)$, Distribute $(-4)(3x)+(-4)(4) = -12x + (-16) = -12x - 16$

Dividing an Expression

For all real numbers a, b and c, where c≠0:

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c} \ and \ \frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$$

Example

$$\frac{2x^3 - 6x}{2x} = \frac{2x^3}{2x} - \frac{6x}{2x} = x^2 - 3$$